COMMUNICATIONS-2 (GATE - 2021) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(17) CORRECT(16) INCORRECT(0) SKIPPED(1)

Q. 1 ? FAQ ▶ Solution Video Have any Doubt ?

If X is a random variable which is uniformly distributed in the interval [0, K], then the value of $E[X^{K-1}]$ is equal to

A $(K)^K$

B $(K)^{K-1}$

C $(K)^{K-2}$ Your answer is Correct

Solution :
(c)

The amplitude of uniformly distributed is equal to $\frac{1}{K}$.

$$\therefore \quad E[X^{K-1}] = \int_{-\infty}^{\infty} X^{K-1} f_x(x)dx$$

$$= \frac{1}{K}\int_0^k x^{K-1}dx = \frac{1}{K}\left|\frac{x^K}{K}\right|_0^K = (K)^{K-2}$$

D K

QUESTION ANALYTICS +

Q. 2 ▶ Solution Video Have any Doubt ?

For discrete memoryless source 'X' with two symbol x_1 and x_2 are $P(x_2) = 0.9$ and $P(x_1) = 0.1$. Then the value $H(X^2)$ is equal to

A 0.938 Your answer is Correct

Solution :
(a)

$$H(X) = 0.9\log_2\frac{1}{0.9} + 0.1\log_2\frac{1}{0.1} = 0.469 \text{ bits/message}$$

$$\therefore \quad H(X^2) = 2H(x) = 2 \times 0.469 = 0.938 \text{ bits/message}$$

B 0.635

C 0.425

D 0.255

QUESTION ANALYTICS +

Q. 3 ▶ Solution Video Have any Doubt ?

The probability density function of a random variable 'X' is given as $f_X(x)$. A random variable Y is defined as $Y = 2X$. The PDF of random variable 'Y' is defined as $kf_X\left(\frac{y}{2}\right)$, then the value of k is equal to

A $\frac{1}{2}$ Your answer is Correct

Solution :
(a)

Since, $\quad Y = 2x$

Thus, $\quad \frac{dx}{dy} = \frac{1}{2}$

$$f_Y(y) = \frac{dx}{dy} f_X(x) = \frac{dx}{dy} f_X\left(\frac{y}{2}\right)$$

$$\therefore \quad K = \frac{1}{2}$$

B 1

C 2

D $\frac{1}{4}$

QUESTION ANALYTICS +

Q. 4 ? FAQ ▶ Solution Video Have any Doubt ?

Which of the following is a valid power spectral density?

A $j\omega^2 + 5$

B $6\delta(\omega)$ Your answer is Correct

Solution :
(b)

C $\frac{4}{4+5\omega}$

D $\frac{e^{-|\omega|}}{1+\omega+\omega^2}$

QUESTION ANALYTICS +

Q. 5 ▶ Solution Video Have any Doubt ?

A source transmits two independent messages with probabilities of P and $(1 - P)$ respectively.
The maximum entropy is achieved when P is equal to ______.

A 0.1

B 0.25

C 0.5 Your answer is Correct

Solution :
(c)

$$H = P\log_2\frac{1}{P} + (1-P)\log_2\frac{1}{1-P}$$

$$\frac{dH}{dp} = 0$$

Which gives as $\quad P = \frac{1}{2}$

D 0.75

QUESTION ANALYTICS +

Q. 6 ? FAQ ▶ Solution Video Have any Doubt ?

Let 'X' be random variable with mean 5 and variance 3. A new random variable 'Y' is defined as Y = 5X + 9, the variance of Y is equal to ______.

75 Your answer is Correct75

Solution :
75

$$\sigma_Y^2 = (5)^2 \times \sigma_X^2 = 25 \times 3 = 75$$

QUESTION ANALYTICS +

Q. 7 ? FAQ ▶ Solution Video Have any Doubt ?

Consider the power spectral density of a random process $X(t)$ is given by
$S_X(\omega) = 5\delta(\omega) + 1.5\delta(\omega - 2\pi) + 1.5\delta(\omega + 2\pi)$
Then, the average power of the process is equal to _____ W.

1.27 (1.25 - 1.30) Your answer is Correct1.27

Solution :
1.27 (1.25 - 1.30)

$$P = E[X^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega)d\omega$$

$$= \frac{1}{2\pi}[5 + 1.5 + 1.5] = \frac{8}{2\pi} = 1.27$$

QUESTION ANALYTICS +

Q. 8 ? FAQ ▶ Solution Video Have any Doubt ?

A continuous random variable X has a pdf

$$f_X(x) = \begin{cases} \frac{1}{M} & ; \quad 0 \le x \le M \\ 0 & ; \quad \text{Otherwise} \end{cases}$$

Then the value of differential entropy

A increases with increase in value of M. Your option is Correct

B decreases with increase value of M.

C is equal to $\log_2 M$. Your option is Correct

D it is independent of M.

YOUR ANSWER - a,c | CORRECT ANSWER - a,c | STATUS - ✓

Solution :
(a, c)

$$H(x) = -\int_{-\infty}^{\infty} f_X(x)\cdot\log_2 f_X(x)dx$$

$$= -\int_0^M \frac{1}{M}\log_2\frac{1}{M}dx = \log_2 M$$

QUESTION ANALYTICS +

Q. 9 ▶ Solution Video Have any Doubt ?

Which of the following is a property of mutual information $I(X ; Y)$ where X and Y are two random variables.

A $I(X ; Y) = I(Y ; X)$ Your option is Correct

B $I(X ; Y) \neq 0$

C $I(X ; Y) = H(X) + H(Y) - H(X, Y)$ Your option is Correct

D $I(X ; Y) = H(Y) - H\left(\frac{Y}{X}\right)$ Your option is Correct

YOUR ANSWER - a,c,d | CORRECT ANSWER - a,c,d | STATUS - ✓

Solution :
(a, c, d)
$I(X ; Y)$ is always non-negative i.e. $I(X ; Y) \ge 0$, but it can be equal to zero.

QUESTION ANALYTICS +

Q. 10 ? FAQ ▶ Solution Video Have any Doubt ?

A random process is defined as $X(t) = 6e^{At}$, where 'A' is random variable uniformly distributed in the interval [0, 2]. Then the autocorrelation function $R_X(t_1, t_2)$ is given as

A $18[e^{2(t_1+t_2)} - 1]$

B $18[e^{2(t_1-t_2)} - 1]$

C $\frac{18}{t_1+t_2}[e^{2(t_1+t_2)} - 1]$ Your answer is Correct

Solution :
(c)

$$X(t) = 6e^{At}$$

$$R_X(t_1, t_2) = E[X(t_1)X(t_2)] = E[6e^{At_1}\, 6e^{At_2}]$$

$$= 36\left[\frac{1}{2}\int_0^2 e^{A(t_1+t_2)}dA\right] = 18\left[\frac{e^{A(t_1+t_2)}}{t_1+t_2}\right]_0^2 = \frac{18}{t_1+t_2}[e^{2(t_1+t_2)} - 1]$$

D $\frac{18}{t_1-t_2}[e^{2(t_1-t_2)} - 1]$

QUESTION ANALYTICS +

Item 1-10 of 17 « previous 1 2 next »

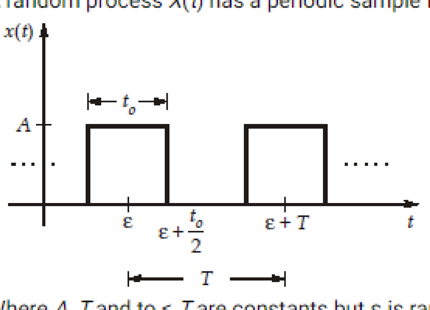
COMMUNICATIONS-2 (GATE - 2021) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(17) CORRECT(16) INCORRECT(0) SKIPPED(1)

Q. 11 ? FAQ | Solution Video | Have any Doubt ?

A random process *X*(*t*) has a periodic sample function as shown in the figure below:



Where *A*, *T* and to ≤ *T* are constants but ε is random variable uniformly distributed on the interval (*0*, *T*), then the pdf is equal to

A $\left(\frac{T-t_o}{T}\right)\delta(x)+\frac{t_o}{T}\delta(x-A)$ — Your answer is Correct

Solution :
(a)
The procedure are identical, let ∈ = *e*,

The above value should be non-negative, this will happen only for $\left(\frac{T-t_o}{T}\right)$ and also since *x* can have only two values *A* and 0, thus

Here, $F_X(x|\varepsilon=e) = P\{X\le x|\varepsilon=e\}$

$$= \left[\frac{(T-t_o)}{T}u(x)+\frac{t_o}{T}u(x-A)\right]$$

Because '*x*' can have only value of zero and *A*.

Thus, $f_X(x|\varepsilon=e) = \left[\frac{(T-t_o)}{T}\right]\delta(x)+\frac{t_o}{T}\delta(x)$

$$f_X(x) = \int_{-\infty}^{\infty} f_X(x|\varepsilon=e)f_\varepsilon(e)de$$

$$= \left[\frac{(T-t_o)}{T}\right]\delta(x)+\left(\frac{t_o}{T}\right)\delta(x-A)$$

B $\frac{t_o}{T}\delta(x)+\left(\frac{T-t_o}{T}\right)\delta(x-A)$

C $\left(\frac{T-t_o}{t_o}\right)\delta(x)+\left(\frac{T-t_o}{T}\right)\delta(x-A)$

D $\frac{t_o}{T}\delta(x)+\frac{T}{t_o}\delta(x-A)$

QUESTION ANALYTICS +

Q. 12 ? FAQ | Solution Video | Have any Doubt ?

The random variable *Y* is defined by $Y=\frac{1}{2}(X+|X|)$ where '*X*' is another random variable. Then the density function of '*Y* for *y* > 0 is equal to

A $f_Y(y)=\frac{f_X(y)}{1-F_X(0)}$ — Your answer is Correct

Solution :
(a)
Given, $y=\frac{1}{2}(x+|x|)$

when $x > 0$, $y = \frac{1}{2}(x+|x|)=\frac{1}{2}(x+x)=\frac{2x}{2}=x,\ y>0$

and $y = x,\ \ y > 0$

∴ $F_Y(y) = P[X\le y|X\ge 0]$

$$= \frac{P[X\le y, X\ge 0]}{P(X\ge 0)}=\frac{P(X\le y, X\ge 0)}{1-P(X<0)}$$

$$= \frac{P(0\le X\le y)}{1-P(X\le 0)}=\frac{F_X(y)-F_X(0)}{1-F_X(0)}$$

∴ $f_Y(y) = \frac{d}{dy}F_Y(y)=\frac{f_X(y)}{1-F_X(0)}$

B $f_Y(y)=\frac{f_Y(x)}{1-F_Y(0)}$

C $f_Y(y)=\frac{f_X(x)}{1-F_X(0)}$

D $f_Y(y)=\frac{f_X(x)}{1+F_Y(0)}$

QUESTION ANALYTICS +

Q. 13 Solution Video | Have any Doubt ?

A random variable *X* has the PDF $f_X(x)$ given by

$$f_X(x) = \begin{cases} Ce^{-x} & ; \ x>0 \\ 0 & ; \ x\le 0\end{cases}$$

Then the value of *C* is equal to

A 1 — Your answer is Correct

Solution :
(a)

$$\int_{-\infty}^{\infty} f_X(x)dx = 1$$

$$\int_0^{\infty} Cxe^{-x}dx = 1$$

$$C\left[x\left(\frac{e^{-x}}{-1}\right)-1\left(\frac{e^{-x}}{1}\right)\right]_0^{\infty} = 1$$

$$C(0 + 1) = 1$$

∴ $C = 1$

B 2

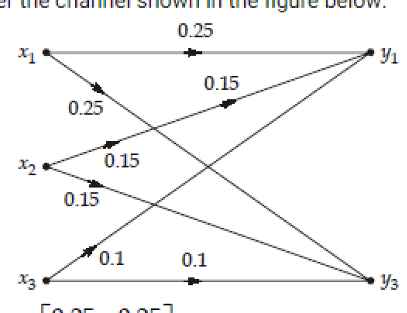
C 3

D 4

QUESTION ANALYTICS +

Q. 14 ? FAQ | Solution Video | Have any Doubt ?

Consider the channel shown in the figure below:



If $P(X, Y) = \begin{bmatrix}0.25 & 0.25\\0.15 & 0.15\\0.1 & 0.1\end{bmatrix}$ then the mutual information is equal to $I(X\,;\,Y)$ = ______.

0 — Correct Option

Solution :
0

$$P(X, Y) = \begin{bmatrix}0.25 & 0.25\\0.15 & 0.15\\0.1 & 0.1\end{bmatrix}$$

$P(x_1) = 0.25 + 0.25 = 0.5$
$P(x_2) = 0.15 + 0.15 = 0.3$
$P(x_3) = 0.1 + 0.1 = 0.2$
and $P(y_1) = 0.5$
$P(y_2) = 0.5$

$$P(X) = P(x_1)\log_2\frac{1}{P(x_1)}+P(x_2)\log_2\frac{1}{P(x_2)}+P(x_3)\log_2\frac{1}{P(x_3)}$$

$$= 0.5\log_2\frac{1}{0.5}+0.3\log_2\frac{1}{0.3}+0.2\log_2\frac{1}{0.2}$$

= 1.485 bit/symbol

and, $H(Y) = P(y_1)\log_2\frac{1}{P(y_1)}+P(y_2)\log_2\frac{1}{P(y_2)}$

$$= 0.5\log_2\frac{1}{0.5}+0.5\log_2\frac{1}{0.5}=1\text{ bit/symbol}$$

$$H(X, Y) = 0.25\log_2\frac{1}{0.25}+0.25\log_2\frac{1}{0.25}+0.15\log\frac{1}{0.15}+0.15\log\frac{1}{0.15}+0.1\log_2\frac{1}{0.1}+0.1\log\frac{1}{0.1}$$

= 2.485 bits/symbol

$I(X, Y) = H(X) + H(Y) – H(X, Y)$

= 1.485 + 1 – 2.485 = 0 bits/symbol

QUESTION ANALYTICS +

Q. 15 ? FAQ | Solution Video | Have any Doubt ?

The probability density function of a random variable *X* is given by

$$f_X(x) = \frac{8}{x^3} \text{ for } x > 2$$

Then the value of $E\left[\frac{X}{3}\right]$ is equal to ______.

1.333 (1.200 - 1.400) — Your answer is Correct1.33

Solution :
1.333 (1.200 - 1.400)

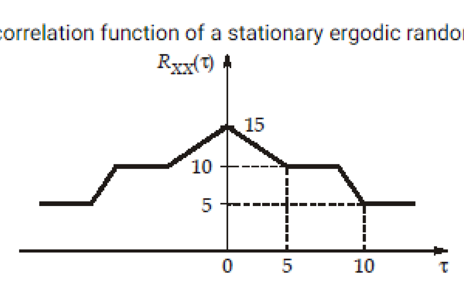
$$E\left[\frac{X}{3}\right] = \int_2^{\infty}\frac{x}{3}f_X(x)dx$$

$$= \int_2^{\infty}\frac{x}{3}\cdot\frac{8}{x^3}dx=\frac{4}{3}=1.33$$

QUESTION ANALYTICS +

Q. 16 ? FAQ | Solution Video | Have any Doubt ?

An autocorrelation function of a stationary ergodic random process is shown below:



Then,

A the mean value *E*[*X*] of the random process is 5.

B the value of total power of the signal is 15. — Your option is Correct

C the value of AC power of the signal is 10. — Your option is Correct

D the value of DC power of the signal is 25.

YOUR ANSWER - b,c | CORRECT ANSWER - b,c | STATUS - ✓

Solution :
(b, c)

$\sigma_x^2 = E[X^2]-\{E[X]\}^2$

$E[X^2(t)] = R_X(0) = 15$

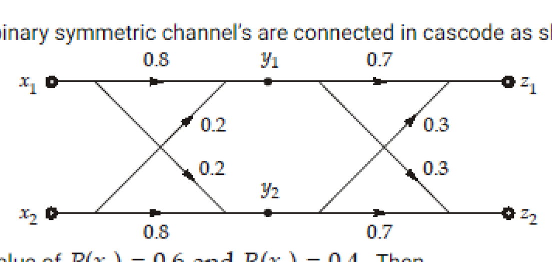
and $E[X(t)]^2 = 5$

∴ $\sigma_x^2 = 15 – 5 = 10$

QUESTION ANALYTICS +

Q. 17 Solution Video | Have any Doubt ?

Two binary symmetric channel's are connected in cascode as shown in the figure below:



The value of $P(x_1) = 0.6$ and $P(x_2) = 0.4$. Then,

A the probability of $P(z_1)$ is equal to 0.524. — Your option is Correct

B the probability of $P(z_2)$ is equal to 0.476. — Your option is Correct

C the probability of $P(y_1) = P(y_2)$.

D If $P(x_1) = P(x_2)$, then $P(z_1) = P(z_2)$. — Your option is Correct

YOUR ANSWER - a,b,d | CORRECT ANSWER - a,b,d | STATUS - ✓

Solution :
(a, b, d)

$$P[Z|X] = \begin{bmatrix}0.8 & 0.2\\0.2 & 0.8\end{bmatrix}\begin{bmatrix}0.7 & 0.3\\0.3 & 0.7\end{bmatrix}=\begin{bmatrix}0.62 & 0.38\\0.38 & 0.62\end{bmatrix}$$

$$P(Z) = [P(x_1)\ P(x_2)]\begin{bmatrix}0.62 & 0.38\\0.38 & 0.62\end{bmatrix}$$

$$= [0.6\ \ 0.4]\begin{bmatrix}0.62 & 0.38\\0.38 & 0.62\end{bmatrix}$$

$P(Z) = [0.524\ \ 0.476]$

∴ $P(Z_1) = 0.524$

If $P(x_1) = P(x_2)$ then

$$P(Z) = [0.5\ 0.5]\begin{bmatrix}0.62 & 0.38\\0.38 & 0.62\end{bmatrix}$$

$P(Z) = [0.5\ \ 0.5]$

QUESTION ANALYTICS +

Item 11-17 of 17 « previous 1 2 next »